Course guide
230857 - NMC - Numerical Methods for Continuum Systems

Last modified: 24/05/2022

Unit in charge: Barcelona School of Telecommunications Engineering
Teaching unit: 748 - FIS - Department of Physics.

Degree: MASTER'S DEGREE IN ENGINEERING PHYSICS (Syllabus 2018). (Optional subject).

Academic year: 2022 **ECTS Credits:** 4.0 **Languages:** English

LECTURER

Coordinating lecturer: Consultar aquí / See here:
https://telecos.upc.edu/ca/estudis/curs-actual/professorat-responsables-coordinadors/responsables-assignatura

Others: Consultar aquí / See here:
https://telecos.upc.edu/ca/estudis/curs-actual/professorat-responsables-coordinadors/professorat-assignat-idioma

PRIOR SKILLS

Knowledge at the level of a degree of Linear Algebra, Calculus, Differential Equations, Numerical Methods and General Physics.

DEGREE COMPETENCES TO WHICH THE SUBJECT CONTRIBUTES

Basic:
CB6. (ENG) Poseer y comprender conocimientos que aporten una base u oportunidad de ser originales en el desarrollo y/o aplicación de ideas, a menudo en un contexto de investigación
CB7. (ENG) Que los estudiantes sepan aplicar los conocimientos adquiridos y su capacidad de resolución de problemas en entornos nuevos o poco conocidos dentro de contextos más amplios (o multidisciplinares) relacionados con su área de estudio.
CB8. (ENG) Que los estudiantes sean capaces de integrar conocimientos y enfrentarse a la complejidad de formular juicios a partir de una información que, siendo incompleta o limitada, incluya reflexiones sobre las responsabilidades sociales y éticas vinculadas a la aplicación de sus conocimientos y juicio.

TEACHING METHODOLOGY

- Lectures.
- Problem solving or programming exercises sessions.
- Assignments to solve exercises or to code numerical methods.
- Programming sessions of numerical methods.

LEARNING OBJECTIVES OF THE SUBJECT

To learn the fundamentals, implementation and the applications of the numerical methods for the partial differential equations of the Mathematical Physics; in particular of the Finite Element methods.

After the course the student must be able to:
- Write the weak form of the differential equations.
- Mesh the computational domain.
- Choose an adequate type of Finite Element for a particular problem.
- Complete the discretization of the problem.
- Write an efficient code to solve the problem.
- Interpret the results and estimate the error of the solution.
- Use a standard Finite Elements library.

STUDY LOAD

Type	Hours	Percentage
Self study	64,0	64.00
Hours large group	36,0	36.00

Total learning time: 100 h

CONTENTS

Introduction.

Description:
In this brief introduction the different methods to discretize the equations of Mathematical Physics will be exposed: finite differences, finite elements, and finite volumes.

Full-or-part-time: 10h
Theory classes: 4h
Self study : 6h

Weak formulation of differential equations, Galerkin and collocation methods.

Description:
It will be explained how to obtain the weak formulation of differential equations, used in the finite elements method, and the different ways to obtain the approximated equations. The weak formulation of several Physics equations (Thermodynamics, Elasticity, Fluid Mechanics, Electromagnetism, etc.) will be written. The Galerkin, Petrov-Galerkin and collocation projections of several equations of Physics will be described.

Full-or-part-time: 7h 30m
Theory classes: 2h
Practical classes: 4h 30m
Self study : 1h

The Finite Element method.

Description:
The objective of this chapter is to introduce the different types of finite elements. It will cover the nodal and modal formulations, the piece-wise Lagrangian approximation in triangles and quadrilaterals, the isoparametric mapping, other types of elements with higher continuity across elements, or other requirements, needed for particular problems. The interpolation errors, and the concepts of h, p, and hp convergence will also be studied.

Full-or-part-time: 10h
Theory classes: 4h
Self study : 6h

Implementation of the Finite Element Method.

Description:
The practical implementation of the FEM will be studied in this chapter, in order to write efficient numerical codes . This includes knowing how to mesh a domain, using for instance open source grid generators as Triangle, Distmesh, Mesh2D or Gmsh, the assembly of the matrices and vectors associated with the linear operators and forcing terms in the equations, using or not quadrature formulas, and estimating the error of the final solutions in some examples. Examples of application developing the full code from scratch in a programming language as Matlab to facilitate the graphical representation or using a high-level FEM library as FEniCS will be studied in detail.

Full-or-part-time: 20h
Theory classes: 4h
Practical classes: 4h
Self study : 12h

The variational theory of the finite element method.

Description:
Introduction to the Functional Analysis tools required to justify the finite element method. Sobolev Spaces. Variational formulation of elliptic problems. Lax-Milgram thorem. Céa's lemma. This chapter will be explained depending on the interests of the students of the course.

Full-or-part-time: 15h
Theory classes: 4h
Practical classes: 2h
Self study : 9h

Time integration.

Description:
The solution of time evolution problems (advection-diffusion, wave equations, Navier-Stokes, etc.) will be studied with special atention to the stability of the numerical time-stepping schemes. Schemes of total discretization, method of lines, operator splitting, etc. will be considered. This chapter will be integrated into the rest of them to optimize the time of the subject.

Full-or-part-time: 7h 30m
Theory classes: 3h
Self study : 4h 30m

High order methods.

Description:
This chapter is an introduction to the spectral elements method which allows reaching a high level of accuracy in space, and the time-stepping algorithms which allow the same in time. The spectral elements can be introduced in the chapter of the FEM without the need of a specific chapter, if details can be avoided. The same holds for the high-order time steppers, which can be seen as particular types of lines methods.

Full-or-part-time: 10h
Theory classes: 4h
Self study : 6h

The finite element method for Navier-Stokes equations.

Description:
Introduction to the formulations of the Navier-Stokes equations for the application of the finite element method. Introduction of the saddle point problems and of the stable elements. Introduction to the steady problems and to the integration via the solution of Stokes problems or the projection methods.

Full-or-part-time: 15h
Theory classes: 5h
Practical classes: 1h
Self study : 9h

Finite elements libraries. Introduction to FEniCS-Python.

Description:
This chapter is optional and will be presented depending on the available time or substituying another chapter. The use of the FEM library FEniCS-Python will be explained with special emphasis in the application to several Physics problems. The students will have to present individual or small group assignments using this library.

Introduction to finite volumes and discontinuous Galerkin methods.

Description:
The limitations of the FEM for the solution of advection-diffusion problems at high Peclet number, for the treatment of hyperbolic equations, etc. will be exposed together with their possible solution by means of the methods giving name to this chapter. This is a complementary chapter.

Complements of Numerical Linear Algebra and of non-linear systems of equations.

Description:
Depending on the previous knowledge of the students it will be necessary to spend some time describing some numerical techniques of Linear Algebra. In particular on
matrix storage for sparse matrices, and computational methods for high-dimensional linear systems and eigenvalue problems. The solution of nonlinear systems of equations, and the study of the dependence of the solutions with the parameters of the problem will also be treated.

Full-or-part-time: 5h
Theory classes: 2h
Self study : 3h

GRADING SYSTEM

- Evaluation of the programming assignments (with possible oral presentation) (PA): 40% + 40% of the final mark.
- Evaluation of home exercises (with possible oral presentation) (PS): 20% of the final mark.
- In case of a continuous lack of attendance to the classes or not delivering the assignments, a final exam of the contents of the subject with a value of 100% of the total mark.
- None of these acts of evaluation will be re-evaluated.

EXAMINATION RULES.

Delivering the exercices and programming assignments is compulsory. It is also attending the classes. Otherwise the final mark will be decided with a single final exam.

BIBLIOGRAPHY

Basic:
- Johnson, C. Numerical solution of partial differential equations by the finite element method. Mineola, NY: Dover Publications, 2009. ISBN 9780486469003.
- Langtangen, H.P.; Logg, A. Solving PDEs in Python: the FEniCS tutorial I. Cham: Springer, 2016. ISBN 9783319524610.
- Larson, M.G.; Bengzon, F. The finite element method: theory, implementation and applications. Berlin: Springer, 2013. ISBN 9783642332869.
- Gockenbach, M.S. Understanding and implementing the finite element method. Philadelphia: SIAM, Society for Industrial and Applied Mathematics, 2006. ISBN 9780898716146.

Complementary:
- Braess, D. Finite elements: theory, fast solvers, and applications in solid mechanics. Cambridge: Cambridge University Press, 2007. ISBN 9780521705189.
- Zienkiewicz, O.C.; Taylor, R.L.; Zhu, J.Z. The finite element method: its basis & fundamentals. 7th ed. Amsterdam: Elsevier Butterworth-Heinemann, 2013. ISBN 9781856176330.
- Zienkiewicz, O.C.; Morgan, K. Finite elements and approximation. Mineola, NY: Dover publications, 1983. ISBN 978-0486453019.
- Elman, H.C.; Silvester, D.J.; Wathen, A.J. Finite elements and fast iterative solvers: with applications in incompressible fluid dynamics. 2nd ed. Oxford: Oxford University Press, 2014. ISBN 9780199678808.

RESOURCES

Audiovisual material:
- Software del curs.. Meshing software and finite element libraries. Proghramming examples. Available at the web page of the subject at Atenea.
- Transparències de la teoria de l'assignatura i llista d'exercicis.. Slides of the full theory of the subject, and list of exercises to solve. Available at the web page of the subject in Atenea.